

Exercise 7.11

Question 1:

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

Answer

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2 \left(\frac{\pi}{2} - x \right) dx \quad \left(\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x) \, dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 2:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$$

Answer

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 3:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 4:

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 5:

$$\int_{-5}^5 |x+2| dx$$

Answer

$$\text{Let } I = \int_{-5}^5 |x+2| dx$$

It can be seen that $(x + 2) \leq 0$ on $[-5, -2]$ and $(x + 2) \geq 0$ on $[-2, 5]$.

$$\begin{aligned} \therefore I &= \int_{-5}^{-2} -(x+2)dx + \int_{-2}^5 (x+2)dx && \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x) \right) \\ I &= - \left[\frac{x^2}{2} + 2x \right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x \right]_{-2}^5 \\ &= - \left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5) \right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2) \right] \\ &= - \left[2 - 4 - \frac{25}{2} + 10 \right] + \left[\frac{25}{2} + 10 - 2 + 4 \right] \\ &= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4 \\ &= 29 \end{aligned}$$

Question 6:

$$\int_2^8 |x-5| dx$$

Answer

$$\text{Let } I = \int_2^8 |x-5| dx$$

It can be seen that $(x-5) \leq 0$ on $[2, 5]$ and $(x-5) \geq 0$ on $[5, 8]$.

$$\begin{aligned} I &= \int_2^5 -(x-5)dx + \int_5^8 (x-5)dx && \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x) \right) \\ &= - \left[\frac{x^2}{2} - 5x \right]_2^5 + \left[\frac{x^2}{2} - 5x \right]_5^8 \\ &= - \left[\frac{25}{2} - 25 - 2 + 10 \right] + \left[32 - 40 - \frac{25}{2} + 25 \right] \\ &= 9 \end{aligned}$$

Question 7:

$$\int_0^1 x(1-x)^n dx$$

Answer

$$\text{Let } I = \int_0^1 x(1-x)^n dx$$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n dx$$

$$= \int_0^1 (1-x)(x)^n dx$$

$$= \int_0^1 (x^n - x^{n+1}) dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2} \right]$$

$$= \frac{(n+2) - (n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

$$\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

Question 8:

$$\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \log (1 + \tan x) dx \quad \dots(1)$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \right\} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \frac{1 - \tan x}{1 + \tan x} \right\} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \frac{2}{(1 + \tan x)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2 dx - \int_0^{\frac{\pi}{4}} \log (1 + \tan x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2 dx - I \quad [\text{From (1)}]$$

$$\Rightarrow 2I = [x \log 2]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

Question 9:

$$\int_0^2 x\sqrt{2-x} dx$$

Answer

$$\text{Let } I = \int_0^2 x\sqrt{2-x} dx$$

$$I = \int_0^2 (2-x)\sqrt{x} dx$$

$$= \int_0^2 \left\{ 2x^{\frac{1}{2}} - x^{\frac{3}{2}} \right\} dx$$

$$= \left[2 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2$$

$$= \left[\frac{4}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} \right]_0^2$$

$$= \frac{4}{3} (2)^{\frac{3}{2}} - \frac{2}{5} (2)^{\frac{5}{2}}$$

$$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

$$\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

Question 10:

$$\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$$

Answer

$$\begin{aligned}
 \text{Let } I &= \int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x) dx \\
 \Rightarrow I &= \int_0^{\frac{\pi}{2}} \{2\log \sin x - \log(2 \sin x \cos x)\} dx \\
 \Rightarrow I &= \int_0^{\frac{\pi}{2}} \{2\log \sin x - \log \sin x - \log \cos x - \log 2\} dx \\
 \Rightarrow I &= \int_0^{\frac{\pi}{2}} \{\log \sin x - \log \cos x - \log 2\} dx \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 \text{It is known that, } \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right) \\
 \Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \cos x - \log \sin x - \log 2\} dx \quad \dots(2)
 \end{aligned}$$

Adding (1) and (2), we obtain

$$\begin{aligned}
 2I &= \int_0^{\frac{\pi}{2}} (-\log 2 - \log 2) dx \\
 \Rightarrow 2I &= -2 \log 2 \int_0^{\frac{\pi}{2}} 1 dx \\
 \Rightarrow I &= -\log 2 \left[\frac{\pi}{2} \right] \\
 \Rightarrow I &= \frac{\pi}{2} (-\log 2) \\
 \Rightarrow I &= \frac{\pi}{2} \left[\log \frac{1}{2} \right] \\
 \Rightarrow I &= \frac{\pi}{2} \log \frac{1}{2}
 \end{aligned}$$

Question 11:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

Answer

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

$$\begin{aligned} I &= 2 \int_0^{\frac{\pi}{2}} \sin^2 x dx \\ &= 2 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx \\ &= \int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx \\ &= \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} \end{aligned}$$

Question 12:

$$\int_0^{\pi} \frac{x dx}{1 + \sin x}$$

Answer

$$\text{Let } I = \int_0^{\pi} \frac{x dx}{1 + \sin x} \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a - x) dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$\begin{aligned}
 2I &= \int_0^{\pi} \frac{\pi}{1 + \sin x} dx \\
 \Rightarrow 2I &= \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx \\
 \Rightarrow 2I &= \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx \\
 \Rightarrow 2I &= \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} dx \\
 \Rightarrow 2I &= \pi [\tan x - \sec x]_0^{\pi} \\
 \Rightarrow 2I &= \pi [2] \\
 \Rightarrow I &= \pi
 \end{aligned}$$

Question 13:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$$

Answer

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx \quad \dots(1)$$

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^7 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^a f(x) dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$$

Question 14:

$$\int_0^{2\pi} \cos^5 x dx$$

Answer

$$\text{Let } I = \int_0^{2\pi} \cos^5 x dx \quad \dots(1)$$

$$\cos^5(2\pi - x) = \cos^5 x$$

It is known that,

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(2a-x) = f(x)$$

$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2 \int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \quad [\cos^5(\pi-x) = -\cos^5 x]$$

Question 15:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right) - \cos\left(\frac{\pi}{2}-x\right)}{1 + \sin\left(\frac{\pi}{2}-x\right) \cos\left(\frac{\pi}{2}-x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

Question 16:

$$\int_0^{\pi} \log(1 + \cos x) dx$$

Answer

$$\text{Let } I = \int_0^{\pi} \log(1 + \cos x) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi-x)) dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \{\log(1 + \cos x) + \log(1 - \cos x)\} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log(1 - \cos^2 x) dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log \sin^2 x dx$$

$$\Rightarrow 2I = 2 \int_0^{\pi} \log \sin x dx$$

$$\Rightarrow I = \int_0^{\pi} \log \sin x dx \quad \dots(3)$$

$$\sin(\pi - x) = \sin x$$

$$\therefore I = 2 \int_0^{\frac{\pi}{2}} \log \sin x dx \quad \dots(4)$$

$$\Rightarrow I = 2 \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = 2 \int_0^{\frac{\pi}{2}} \log \cos x dx \quad \dots(5)$$

Adding (4) and (5), we obtain

$$2I = 2 \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x dx - \int_0^{\frac{\pi}{2}} \log 2 dx$$

$$\text{Let } 2x = t \quad \square \quad 2dx = dt$$

$$\text{When } x = 0, t = 0 \text{ and when } x = \frac{\pi}{2}, t = \pi$$

$$\therefore I = \frac{1}{2} \int_0^{\pi} \log \sin t dt - \frac{\pi}{2} \log 2$$

$$\Rightarrow I = \frac{1}{2} \int_0^{\pi} \log \sin t dt - \frac{\pi}{2} \log 2$$

$$\Rightarrow \frac{I}{2} = -\frac{\pi}{2} \log 2$$

$$\Rightarrow I = -\pi \log 2$$

Question 17:

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$$

Answer

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \dots(1)$$

It is known that, $\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\Rightarrow 2I = \int_0^a 1 dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Question 18:

$$\int_0^4 |x-1| dx$$

Answer

$$I = \int_0^4 |x-1| dx$$

It can be seen that, $(x - 1) \leq 0$ when $0 \leq x \leq 1$ and $(x - 1) \geq 0$ when $1 \leq x \leq 4$

$$\begin{aligned}
 I &= \int_0^1 |x-1| dx + \int_1^4 |x-1| dx && \left(\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx \right) \\
 &= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx \\
 &= \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4 \\
 &= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1 \\
 &= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 \\
 &= 5
 \end{aligned}$$

Question 19:

Show that $\int_0^a f(x)g(x) dx = 2 \int_0^a f(x) dx$, if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

Answer

$$\text{Let } I = \int_0^a f(x)g(x) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^a f(a-x)g(a-x) dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^a f(x)g(a-x) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \{f(x)g(x) + f(x)g(a-x)\} dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{g(x) + g(a-x)\} dx$$

$$\Rightarrow 2I = \int_0^a f(x) \times 4 dx \quad [g(x) + g(a-x) = 4]$$

$$\Rightarrow I = 2 \int_0^a f(x) dx$$

Question 20:

The value of $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx$ is

- A. 0
- B. 2
- C. π
- D. 1

Answer

$$\text{Let } I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx$$

$$\Rightarrow I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx$$

It is known that if $f(x)$ is an even function, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ and

if $f(x)$ is an odd function, then $\int_{-a}^a f(x) dx = 0$

$$I = 0 + 0 + 0 + 2 \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$= 2[x]_0^{\frac{\pi}{2}}$$

$$= \frac{2\pi}{2}$$

$$\pi$$

Hence, the correct Answer is C.

Question 21:

The value of $\int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \right) dx$ is

- A. 2
- B. $\frac{3}{4}$
- C. 0
- D. -2

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \right) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left[\frac{4 + 3 \sin \left(\frac{\pi}{2} - x \right)}{4 + 3 \cos \left(\frac{\pi}{2} - x \right)} \right] dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3 \cos x}{4 + 3 \sin x} \right) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{ \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \right) + \log \left(\frac{4 + 3 \cos x}{4 + 3 \sin x} \right) \right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \times \frac{4 + 3 \cos x}{4 + 3 \sin x} \right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0 dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.