

Exercise 9.4**Question 1:**

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

Answer

The given differential equation is:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \tan^2 \frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \left(\sec^2 \frac{x}{2} - 1 \right)$$

Separating the variables, we get:

$$dy = \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

Now, integrating both sides of this equation, we get:

$$\int dy = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx = \int \sec^2 \frac{x}{2} dx - \int dx$$

$$\Rightarrow y = 2 \tan \frac{x}{2} - x + C$$

This is the required general solution of the given differential equation.

Question 2:

$$\frac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$$

Answer

The given differential equation is:

$$\frac{dy}{dx} = \sqrt{4-y^2}$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{\sqrt{4-y^2}} = dx$$

Now, integrating both sides of this equation, we get:

$$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$\Rightarrow \sin^{-1} \frac{y}{2} = x + C$$

$$\Rightarrow \frac{y}{2} = \sin(x + C)$$

$$\Rightarrow y = 2 \sin(x + C)$$

This is the required general solution of the given differential equation.

Question 3:

$$\frac{dy}{dx} + y = 1 \quad (y \neq 1)$$

Answer

The given differential equation is:

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow dy + y \, dx = dx$$

$$\Rightarrow dy = (1-y) \, dx$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{1-y} = dx$$

Now, integrating both sides, we get:

$$\int \frac{dy}{1-y} = \int dx$$

$$\Rightarrow \log(1-y) = x + \log C$$

$$\Rightarrow -\log C - \log(1-y) = x$$

$$\Rightarrow \log C(1-y) = -x$$

$$\Rightarrow C(1-y) = e^{-x}$$

$$\Rightarrow 1-y = \frac{1}{C} e^{-x}$$

$$\Rightarrow y = 1 - \frac{1}{C} e^{-x}$$

$$\Rightarrow y = 1 + Ae^{-x} \text{ (where } A = -\frac{1}{C}\text{)}$$

This is the required general solution of the given differential equation.

Question 4:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

Answer

The given differential equation is:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

$$\Rightarrow \frac{\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy}{\tan x \tan y} = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} \, dx + \frac{\sec^2 y}{\tan y} \, dy = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} \, dx = -\frac{\sec^2 y}{\tan y} \, dy$$

Integrating both sides of this equation, we get:

$$\int \frac{\sec^2 x}{\tan x} \, dx = -\int \frac{\sec^2 y}{\tan y} \, dy \quad \dots(1)$$

Let $\tan x = t$.

$$\therefore \frac{d}{dx}(\tan x) = \frac{dt}{dx}$$

$$\Rightarrow \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec^2 x \, dx = dt$$

$$\begin{aligned} \text{Now, } \int \frac{\sec^2 x}{\tan x} \, dx &= \int \frac{1}{t} \, dt. \\ &= \log t \\ &= \log(\tan x) \end{aligned}$$

$$\text{Similarly, } \int \frac{\sec^2 y}{\tan y} \, dy = \log(\tan y).$$

Substituting these values in equation (1), we get:

$$\log(\tan x) = -\log(\tan y) + \log C$$

$$\Rightarrow \log(\tan x) = \log\left(\frac{C}{\tan y}\right)$$

$$\Rightarrow \tan x = \frac{C}{\tan y}$$

$$\Rightarrow \tan x \tan y = C$$

This is the required general solution of the given differential equation.

Question 5:

$$(e^x + e^{-x}) \, dy - (e^x - e^{-x}) \, dx = 0$$

Answer

The given differential equation is:

$$(e^x + e^{-x}) \, dy - (e^x - e^{-x}) \, dx = 0$$

$$\Rightarrow (e^x + e^{-x}) \, dy = (e^x - e^{-x}) \, dx$$

$$\Rightarrow dy = \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx$$

Integrating both sides of this equation, we get:

$$\int dy = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C$$
$$\Rightarrow y = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C \quad \dots(1)$$

Let $(e^x + e^{-x}) = t$.

Differentiating both sides with respect to x , we get:

$$\frac{d}{dx}(e^x + e^{-x}) = \frac{dt}{dx}$$
$$\Rightarrow e^x - e^{-x} = \frac{dt}{dx}$$
$$\Rightarrow (e^x - e^{-x}) dx = dt$$

Substituting this value in equation (1), we get:

$$y = \int \frac{1}{t} dt + C$$
$$\Rightarrow y = \log(t) + C$$
$$\Rightarrow y = \log(e^x + e^{-x}) + C$$

This is the required general solution of the given differential equation.

Question 6:

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

Answer

The given differential equation is:

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$
$$\Rightarrow \frac{dy}{1+y^2} = (1+x^2) dx$$

Integrating both sides of this equation, we get:

$$\int \frac{dy}{1+y^2} = \int (1+x^2) dx$$
$$\Rightarrow \tan^{-1} y = \int dx + \int x^2 dx$$
$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

This is the required general solution of the given differential equation.

Question 7:

$$y \log y dx - x dy = 0$$

Answer

The given differential equation is:

$$y \log y dx - x dy = 0$$
$$\Rightarrow y \log y dx = x dy$$
$$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x} \quad \dots(1)$$

Let $\log y = t$.

$$\therefore \frac{d}{dy}(\log y) = \frac{dt}{dy}$$
$$\Rightarrow \frac{1}{y} = \frac{dt}{dy}$$
$$\Rightarrow \frac{1}{y} dy = dt$$

Substituting this value in equation (1), we get:

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \log t = \log x + \log C$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{Cx}$$

This is the required general solution of the given differential equation.

Question 8:

$$x^5 \frac{dy}{dx} = -y^5$$

Answer

The given differential equation is:

$$x^5 \frac{dy}{dx} = -y^5$$

$$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$$

$$\Rightarrow \frac{dx}{x^5} + \frac{dy}{y^5} = 0$$

Integrating both sides, we get:

$$\int \frac{dx}{x^5} + \int \frac{dy}{y^5} = k \quad (\text{where } k \text{ is any constant})$$

$$\Rightarrow \int x^{-5} dx + \int y^{-5} dy = k$$

$$\Rightarrow \frac{x^{-4}}{-4} + \frac{y^{-4}}{-4} = k$$

$$\Rightarrow x^{-4} + y^{-4} = -4k$$

$$\Rightarrow x^{-4} + y^{-4} = C \quad (C = -4k)$$

This is the required general solution of the given differential equation.

Question 9:

$$\frac{dy}{dx} = \sin^{-1} x$$

Answer

The given differential equation is:

$$\frac{dy}{dx} = \sin^{-1} x$$

$$\Rightarrow dy = \sin^{-1} x \, dx$$

Integrating both sides, we get:

$$\int dy = \int \sin^{-1} x \, dx$$

$$\Rightarrow y = \int (\sin^{-1} x \cdot 1) \, dx$$

$$\Rightarrow y = \sin^{-1} x \cdot \int (1) \, dx - \int \left[\left(\frac{d}{dx} (\sin^{-1} x) \right) \cdot \int (1) \, dx \right] dx$$

$$\Rightarrow y = \sin^{-1} x \cdot x - \int \left(\frac{1}{\sqrt{1-x^2}} \cdot x \right) dx$$

$$\Rightarrow y = x \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}} dx \quad \dots(1)$$

Let $1-x^2 = t$.

$$\Rightarrow \frac{d}{dx} (1-x^2) = \frac{dt}{dx}$$

$$\Rightarrow -2x = \frac{dt}{dx}$$

$$\Rightarrow x \, dx = -\frac{1}{2} dt$$

Substituting this value in equation (1), we get:

$$y = x \sin^{-1} x + \int \frac{1}{2\sqrt{t}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \int (t)^{-\frac{1}{2}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{t} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{1-x^2} + C$$

This is the required general solution of the given differential equation.

Question 10:

$$e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$$

Answer

The given differential equation is:

$$e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$$

$$(1 - e^x) \sec^2 y dy = -e^x \tan y dx$$

Separating the variables, we get:

$$\frac{\sec^2 y}{\tan y} dy = \frac{-e^x}{1 - e^x} dx$$

Integrating both sides, we get:

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{-e^x}{1 - e^x} dx \quad \dots(1)$$

Let $\tan y = u$.

$$\Rightarrow \frac{d}{dy} (\tan y) = \frac{du}{dy}$$

$$\Rightarrow \sec^2 y = \frac{du}{dy}$$

$$\Rightarrow \sec^2 y dy = du$$

$$\therefore \int \frac{\sec^2 y}{\tan y} dy = \int \frac{du}{u} = \log u = \log (\tan y)$$

Now, let $1 - e^x = t$.

$$\therefore \frac{d}{dx}(1 - e^x) = \frac{dt}{dx}$$

$$\Rightarrow -e^x = \frac{dt}{dx}$$

$$\Rightarrow -e^x dx = dt$$

$$\Rightarrow \int \frac{-e^x}{1 - e^x} dx = \int \frac{dt}{t} = \log t = \log(1 - e^x)$$

Substituting the values of $\int \frac{\sec^2 y}{\tan y} dy$ and $\int \frac{-e^x}{1 - e^x} dx$ in equation (1), we get:

$$\Rightarrow \log(\tan y) = \log(1 - e^x) + \log C$$

$$\Rightarrow \log(\tan y) = \log[C(1 - e^x)]$$

$$\Rightarrow \tan y = C(1 - e^x)$$

This is the required general solution of the given differential equation.

Question 11:

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x; y = 1 \text{ when } x = 0$$

Answer

The given differential equation is:

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}$$

$$\Rightarrow dy = \frac{2x^2 + x}{(x+1)(x^2+1)} dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}. \quad \dots(2)$$

$$\Rightarrow \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{Ax^2 + A + (Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$\Rightarrow 2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$\Rightarrow 2x^2 + x = (A+B)x^2 + (B+C)x + (A+C)$$

Comparing the coefficients of x^2 and x , we get:

$$A + B = 2$$

$$B + C = 1$$

$$A + C = 0$$

Solving these equations, we get:

$$A = \frac{1}{2}, B = \frac{3}{2} \text{ and } C = -\frac{1}{2}$$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{1}{2} \cdot \frac{1}{x+1} + \frac{1}{2} \frac{(3x-1)}{x^2+1}$$

Therefore, equation (1) becomes:

$$\begin{aligned}
 \int dy &= \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{3x-1}{x^2+1} dx \\
 \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx \\
 \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \cdot \int \frac{2x}{x^2+1} dx - \frac{1}{2} \tan^{-1} x + C \\
 \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C \\
 \Rightarrow y &= \frac{1}{4} [2 \log(x+1) + 3 \log(x^2+1)] - \frac{1}{2} \tan^{-1} x + C \\
 \Rightarrow y &= \frac{1}{4} [(x+1)^2 (x^2+1)^3] - \frac{1}{2} \tan^{-1} x + C \quad \dots(3)
 \end{aligned}$$

Now, $y = 1$ when $x = 0$.

$$\Rightarrow 1 = \frac{1}{4} \log(1) - \frac{1}{2} \tan^{-1} 0 + C$$

$$\Rightarrow 1 = \frac{1}{4} \times 0 - \frac{1}{2} \times 0 + C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (3), we get:

$$y = \frac{1}{4} [\log(x+1)^2 (x^2+1)^3] - \frac{1}{2} \tan^{-1} x + 1$$

Question 12:

$$x(x^2-1) \frac{dy}{dx} = 1; \quad y = 0 \text{ when } x = 2$$

Answer

$$x(x^2-1) \frac{dy}{dx} = 1$$

$$\Rightarrow dy = \frac{dx}{x(x^2-1)}$$

$$\Rightarrow dy = \frac{1}{x(x-1)(x+1)} dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{1}{x(x-1)(x+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}. \quad \dots(2)$$

$$\begin{aligned} \Rightarrow \frac{1}{x(x-1)(x+1)} &= \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)} \\ &= \frac{(A+B+C)x^2 + (B-C)x - A}{x(x-1)(x+1)} \end{aligned}$$

Comparing the coefficients of x^2 , x , and constant, we get:

$$A = -1$$

$$B - C = 0$$

$$A + B + C = 0$$

Solving these equations, we get $B = \frac{1}{2}$ and $C = \frac{1}{2}$.

Substituting the values of A , B , and C in equation (2), we get:

$$\frac{1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}$$

Therefore, equation (1) becomes:

$$\int dy = -\int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{1}{x+1} dx$$

$$\Rightarrow y = -\log x + \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1) + \log k$$

$$\Rightarrow y = \frac{1}{2} \log \left[\frac{k^2(x-1)(x+1)}{x^2} \right] \quad \dots(3)$$

Now, $y = 0$ when $x = 2$.

$$\Rightarrow 0 = \frac{1}{2} \log \left[\frac{k^2(2-1)(2+1)}{4} \right]$$

$$\Rightarrow \log \left(\frac{3k^2}{4} \right) = 0$$

$$\Rightarrow \frac{3k^2}{4} = 1$$

$$\Rightarrow 3k^2 = 4$$

$$\Rightarrow k^2 = \frac{4}{3}$$

Substituting the value of k^2 in equation (3), we get:

$$y = \frac{1}{2} \log \left[\frac{4(x-1)(x+1)}{3x^2} \right]$$

$$y = \frac{1}{2} \log \left[\frac{4(x^2-1)}{3x^2} \right]$$

Question 13:

$$\cos \left(\frac{dy}{dx} \right) = a \quad (a \in R); y = 1 \text{ when } x = 0$$

Answer

$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a \, dx$$

Integrating both sides, we get:

$$\int dy = \cos^{-1} a \int dx$$

$$\Rightarrow y = \cos^{-1} a \cdot x + C$$

$$\Rightarrow y = x \cos^{-1} a + C \quad \dots(1)$$

Now, $y = 1$ when $x = 0$.

$$\Rightarrow 1 = 0 \cdot \cos^{-1} a + C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (1), we get:

$$y = x \cos^{-1} a + 1$$

$$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$$

$$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$$

Question 14:

$$\frac{dy}{dx} = y \tan x; y = 1 \text{ when } x = 0$$

Answer

$$\frac{dy}{dx} = y \tan x$$

$$\Rightarrow \frac{dy}{y} = \tan x \, dx$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = - \int \tan x \, dx$$

$$\Rightarrow \log y = \log(\sec x) + \log C$$

$$\Rightarrow \log y = \log(C \sec x)$$

$$\Rightarrow y = C \sec x \quad \dots(1)$$

Now, $y = 1$ when $x = 0$.

$$\Rightarrow 1 = C \times \sec 0$$

$$\Rightarrow 1 = C \times 1$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (1), we get:

$$y = \sec x$$

Question 15:

Find the equation of a curve passing through the point (0, 0) and whose differential

equation is $y' = e^x \sin x$.

Answer

The differential equation of the curve is:

$$y' = e^x \sin x$$

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x \, dx \quad \dots(1)$$

$$\text{Let } I = \int e^x \sin x \, dx.$$

$$\Rightarrow I = \sin x \int e^x \, dx - \int \left(\frac{d}{dx}(\sin x) \cdot \int e^x \, dx \right) dx$$

$$\begin{aligned} \Rightarrow I &= \sin x \cdot e^x - \int \cos x \cdot e^x dx \\ \Rightarrow I &= \sin x \cdot e^x - \left[\cos x \cdot \int e^x dx - \int \left(\frac{d}{dx} (\cos x) \cdot \int e^x dx \right) dx \right] \\ \Rightarrow I &= \sin x \cdot e^x - \left[\cos x \cdot e^x - \int (-\sin x) \cdot e^x dx \right] \\ \Rightarrow I &= e^x \sin x - e^x \cos x - I \\ \Rightarrow 2I &= e^x (\sin x - \cos x) \\ \Rightarrow I &= \frac{e^x (\sin x - \cos x)}{2} \end{aligned}$$

Substituting this value in equation (1), we get:

$$y = \frac{e^x (\sin x - \cos x)}{2} + C \quad \dots(2)$$

Now, the curve passes through point (0, 0).

$$\begin{aligned} \therefore 0 &= \frac{e^0 (\sin 0 - \cos 0)}{2} + C \\ \Rightarrow 0 &= \frac{1(0-1)}{2} + C \\ \Rightarrow C &= \frac{1}{2} \end{aligned}$$

Substituting $C = \frac{1}{2}$ in equation (2), we get:

$$\begin{aligned} y &= \frac{e^x (\sin x - \cos x)}{2} + \frac{1}{2} \\ \Rightarrow 2y &= e^x (\sin x - \cos x) + 1 \\ \Rightarrow 2y - 1 &= e^x (\sin x - \cos x) \end{aligned}$$

Hence, the required equation of the curve is $2y - 1 = e^x (\sin x - \cos x)$.

Question 16:

For the differential equation $xy \frac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through the point (1, -1).

Answer

The differential equation of the given curve is:

$$\begin{aligned} xy \frac{dy}{dx} &= (x+2)(y+2) \\ \Rightarrow \left(\frac{y}{y+2} \right) dy &= \left(\frac{x+2}{x} \right) dx \\ \Rightarrow \left(1 - \frac{2}{y+2} \right) dy &= \left(1 + \frac{2}{x} \right) dx \end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned} \int \left(1 - \frac{2}{y+2} \right) dy &= \int \left(1 + \frac{2}{x} \right) dx \\ \Rightarrow \int dy - 2 \int \frac{1}{y+2} dy &= \int dx + 2 \int \frac{1}{x} dx \\ \Rightarrow y - 2 \log(y+2) &= x + 2 \log x + C \\ \Rightarrow y - x - C &= \log x^2 + \log(y+2)^2 \\ \Rightarrow y - x - C &= \log \left[x^2 (y+2)^2 \right] \quad \dots(1) \end{aligned}$$

Now, the curve passes through point (1, -1).

$$\begin{aligned} \Rightarrow -1 - 1 - C &= \log \left[(1)^2 (-1+2)^2 \right] \\ \Rightarrow -2 - C &= \log 1 = 0 \\ \Rightarrow C &= -2 \end{aligned}$$

Substituting $C = -2$ in equation (1), we get:

$$y - x + 2 = \log \left[x^2 (y+2)^2 \right]$$

This is the required solution of the given curve.

Question 17:

Find the equation of a curve passing through the point (0, -2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y -coordinate of the point is equal to the x -coordinate of the point.

Answer

Let x and y be the x -coordinate and y -coordinate of the curve respectively.

We know that the slope of a tangent to the curve in the coordinate axis is given by the relation,

$$\frac{dy}{dx}$$

According to the given information, we get:

$$y \cdot \frac{dy}{dx} = x$$

$$\Rightarrow y \, dy = x \, dx$$

Integrating both sides, we get:

$$\int y \, dy = \int x \, dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow y^2 - x^2 = 2C \quad \dots(1)$$

Now, the curve passes through point $(0, -2)$.

$$\therefore (-2)^2 - 0^2 = 2C$$

$$\Rightarrow 2C = 4$$

Substituting $2C = 4$ in equation (1), we get:

$$y^2 - x^2 = 4$$

This is the required equation of the curve.

Question 18:

At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2, 1)$.

Answer

It is given that (x, y) is the point of contact of the curve and its tangent.

The slope (m_1) of the line segment joining (x, y) and $(-4, -3)$ is $\frac{y+3}{x+4}$.

We know that the slope of the tangent to the curve is given by the relation,

$$\frac{dy}{dx}$$

$$\therefore \text{Slope } (m_2) \text{ of the tangent} = \frac{dy}{dx}$$

According to the given information:

$$m_2 = 2m_1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(y+3)}{x+4}$$

$$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$$

Integrating both sides, we get:

$$\int \frac{dy}{y+3} = 2 \int \frac{dx}{x+4}$$

$$\Rightarrow \log(y+3) = 2 \log(x+4) + \log C$$

$$\Rightarrow \log(y+3) = \log C(x+4)^2$$

$$\Rightarrow y+3 = C(x+4)^2 \quad \dots(1)$$

This is the general equation of the curve.

It is given that it passes through point $(-2, 1)$.

$$\Rightarrow 1+3 = C(-2+4)^2$$

$$\Rightarrow 4 = 4C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (1), we get:

$$y + 3 = (x + 4)^2$$

This is the required equation of the curve.

Question 19:

The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Answer

Let the rate of change of the volume of the balloon be k (where k is a constant).

$$\Rightarrow \frac{dv}{dt} = k$$

$$\Rightarrow \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \right) = k \quad \left[\text{Volume of sphere} = \frac{4}{3} \pi r^3 \right]$$

$$\Rightarrow \frac{4}{3} \pi \cdot 3r^2 \cdot \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = k dt$$

Integrating both sides, we get:

$$4\pi \int r^2 dr = k \int dt$$

$$\Rightarrow 4\pi \cdot \frac{r^3}{3} = kt + C$$

$$\Rightarrow 4\pi r^3 = 3(kt + C) \quad \dots(1)$$

Now, at $t = 0$, $r = 3$:

$$\Rightarrow 4\pi \times 3^3 = 3(k \times 0 + C)$$

$$\Rightarrow 108\pi = 3C$$

$$\Rightarrow C = 36\pi$$

At $t = 3$, $r = 6$:

$$\Rightarrow 4\pi \times 6^3 = 3(k \times 3 + C)$$

$$\Rightarrow 864\pi = 3(3k + 36\pi)$$

$$\Rightarrow 3k = -288\pi - 36\pi = 252\pi$$

$$\Rightarrow k = 84\pi$$

Substituting the values of k and C in equation (1), we get:

$$4\pi r^3 = 3[84\pi t + 36\pi]$$

$$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after t seconds is $(63t + 27)^{\frac{1}{3}}$.

Question 20:

In a bank, principal increases continuously at the rate of $r\%$ per year. Find the value of r if Rs 100 doubles itself in 10 years ($\log_e 2 = 0.6931$).

Answer

Let p , t , and r represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of $r\%$ per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$

$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{r}{100} \int dt$$

$$\Rightarrow \log p = \frac{rt}{100} + k$$

$$\Rightarrow p = e^{\frac{rt}{100} + k} \quad \dots(1)$$

It is given that when $t = 0$, $p = 100$.

$$\Rightarrow 100 = e^k \quad \dots (2)$$

Now, if $t = 10$, then $p = 2 \times 100 = 200$.

Therefore, equation (1) becomes:

$$200 = e^{\frac{r}{10} + k}$$

$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot e^k$$

$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot 100 \quad \text{(From (2))}$$

$$\Rightarrow e^{\frac{r}{10}} = 2$$

$$\Rightarrow \frac{r}{10} = \log_e 2$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow r = 6.931$$

Hence, the value of r is 6.93%.

Question 21:

In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

Answer

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100}\right)p$$

$$\Rightarrow \frac{dp}{p} = \frac{5}{20} dt$$

$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{1}{20} \int dt$$

$$\Rightarrow \log p = \frac{t}{20} + C$$

$$\Rightarrow p = e^{\frac{t}{20} + C} \quad \dots(1)$$

Now, when $t = 0$, $p = 1000$.

$$\Rightarrow 1000 = e^C \dots (2)$$

At $t = 10$, equation (1) becomes:

$$p = e^{\frac{10}{20} + C}$$

$$\Rightarrow p = e^{0.5} \times e^C$$

$$\Rightarrow p = 1.648 \times 1000$$

$$\Rightarrow p = 1648$$

Hence, after 10 years the amount will worth Rs 1648.

Question 22:

In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Answer

Let y be the number of bacteria at any instant t .

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \propto y$$

$$\Rightarrow \frac{dy}{dt} = ky \text{ (where } k \text{ is a constant)}$$

$$\Rightarrow \frac{dy}{y} = k dt$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = k \int dt$$

$$\Rightarrow \log y = kt + C \quad \dots(1)$$

Let y_0 be the number of bacteria at $t = 0$.

$$\Rightarrow \log y_0 = C$$

Substituting the value of C in equation (1), we get:

$$\log y = kt + \log y_0$$

$$\Rightarrow \log y - \log y_0 = kt$$

$$\Rightarrow \log \left(\frac{y}{y_0} \right) = kt$$

$$\Rightarrow kt = \log \left(\frac{y}{y_0} \right) \quad \dots(2)$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\begin{aligned}\Rightarrow y &= \frac{110}{100} y_0 \\ \Rightarrow \frac{y}{y_0} &= \frac{11}{10} \quad \dots(3)\end{aligned}$$

Substituting this value in equation (2), we get:

$$\begin{aligned}k \cdot 2 &= \log\left(\frac{11}{10}\right) \\ \Rightarrow k &= \frac{1}{2} \log\left(\frac{11}{10}\right)\end{aligned}$$

Therefore, equation (2) becomes:

$$\begin{aligned}\frac{1}{2} \log\left(\frac{11}{10}\right) \cdot t &= \log\left(\frac{y}{y_0}\right) \\ \Rightarrow t &= \frac{2 \log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \quad \dots(4)\end{aligned}$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1 .

$$\Rightarrow y = 2y_0 \text{ at } t = t_1$$

From equation (4), we get:

$$t_1 = \frac{2 \log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Question 23:

The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

A. $e^x + e^{-y} = C$

B. $e^x + e^y = C$

C. $e^{-x} + e^y = C$

D. $e^{-x} + e^{-y} = C$

Answer

$$\frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^{-y} dy = e^x dx$$

Integrating both sides, we get:

$$\int e^{-y} dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + k$$

$$\Rightarrow e^x + e^{-y} = -k$$

$$\Rightarrow e^x + e^{-y} = c \quad (c = -k)$$

Hence, the correct answer is A.