Exercise 9.6

Question 1:

$$\frac{dy}{dx}+2y=\sin x$$

Answer

The given differential equation is $\frac{dy}{dx}+2y=\sin x.$

This is in the form of $\frac{dy}{dx}+py=Q\,(\text{where } p=2 \text{ and } Q=\sin x).$

$$\text{Now, I.F } = e^{\int p\,dx}=e^{\int 2\,dx}=e^{2x}.$$

The solution of the given differential equation is given by the relation,

$$y(\text{I.F.})=\int(\text{Q}\times\text{I.F.})\,dx+\text{C}$$

$$\Rightarrow ye^{2x}=\int\sin x\cdot e^{2x}dx+\text{C} \qquad \ldots(1)$$

$$\text{Let } I=\int\sin x\cdot e^{2x}.$$

$$\Rightarrow I=\sin x\cdot\int e^{2x}dx-\int\left(\frac{d}{dx}(\sin x)\cdot\int e^{2x}dx\right)dx$$

$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\left(\cos x\cdot\frac{e^{2x}}{2}\right)dx$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\cdot\int e^{2x}-\int\left(\frac{d}{dx}(\cos x)\cdot\int e^{2x}dx\right)dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{2}-\int\left[(-\sin x)\cdot\frac{e^{2x}}{2}\right]dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}\int\left(\sin x.e^{2x}\right)dx$$

$$\Rightarrow I=\frac{e^{2x}}{4}(2\sin x-\cos x)-\frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}}{4}(2\sin x-\cos x)$$

$$\Rightarrow I=\frac{e^{2x}}{5}(2\sin x-\cos x)$$

Therefore, equation (1) becomes:

$$ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$$

$$\Rightarrow y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$$

This is the required general solution of the given differential equation.

Question 2:

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Answer

$$\frac{dy}{dx} + py = Q \text{ (where } p = 3 \text{ and } Q = e^{-2x}\text{)}.$$

The given differential equation is

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int 3\,dx} = e^{3x}.$$

The solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int(\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow ye^{3x} = \int\left(e^{-2x} \times e^{3x}\right) + \text{C}$$

$$\Rightarrow ye^{3x} = \int e^{x}\,dx + \text{C}$$

$$\Rightarrow ye^{3x} = e^{x} + \text{C}$$

$$\Rightarrow y = e^{-2x} + \text{C}e^{-3x}$$

This is the required general solution of the given differential equation.

Question 3:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Answer

The given differential equation is:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{1}{x} \text{ and } Q = x^2\text{)}$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x.$$

The solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y(x) = \int (x^2 \cdot x)\,dx + \text{C}$$

$$\Rightarrow xy = \int x^3\,dx + \text{C}$$

$$\Rightarrow xy = \frac{x^4}{4} + \text{C}$$

This is the required general solution of the given differential equation.

Question 4:

$$\frac{dy}{dx} + \sec x y = \tan x\left(0 \le x < \frac{\pi}{2}\right)$$

Answer

The given differential equation is:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \sec x \text{ and } Q = \tan x)$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int \sec x\,dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y(\sec x + \tan x) = \int \tan x(\sec x + \tan x)\,dx + \text{C}$$

$$\Rightarrow y(\sec x + \tan x) = \int \sec x \tan x dx + \int \tan^2 x dx + \text{C}$$

$$\Rightarrow y(\sec x + \tan x) = \sec x + \int (\sec^2 x - 1)\,dx + \text{C}$$

$$\Rightarrow y(\sec x + \tan x) = \sec x + \tan x - x + \text{C}$$

Question 5:

$$\int_0^{\frac{\pi}{2}} \cos 2x\,dx$$

Answer

Let $I = \int_0^{\frac{\pi}{2}} \cos 2x \, dx$

$\int \cos 2x \, dx = \left(\frac{\sin 2x}{2} \right) = \mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$I = \mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)$

$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$

$= \frac{1}{2}\left[\sin \pi - \sin 0\right]$

$= \frac{1}{2}\left[0 - 0\right] = 0$

Question 6:

$x\frac{dy}{dx} + 2y = x^2 \log x$

Answer

The given differential equation is:

$x\frac{dy}{dx} + 2y = x^2 \log x$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x \log x$

This equation is in the form of a linear differential equation as:

$\frac{dy}{dx} + py = Q$ (where $p = \frac{2}{x}$ and $Q = x \log x$)

Now, I.F $= e^{\int p\,dx} = e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2.$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int(\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y \cdot x^2 = \int\left(x\log x \cdot x^2\right)dx + \text{C}$$

$$\Rightarrow x^2 y = \int\left(x^3 \log x\right)dx + \text{C}$$

$$\Rightarrow x^2 y = \log x \cdot \int x^3 dx - \int\left[\frac{d}{dx}(\log x) \cdot \int x^3 dx\right]dx + \text{C}$$

$$\Rightarrow x^2 y = \log x \cdot \frac{x^4}{4} - \int\left(\frac{1}{x} \cdot \frac{x^4}{4}\right)dx + \text{C}$$

$$\Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{4}\int x^3 dx + \text{C}$$

$$\Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + \text{C}$$

$$\Rightarrow x^2 y = \frac{1}{16}x^4\left(4\log x - 1\right) + \text{C}$$

$$\Rightarrow y = \frac{1}{16}x^2\left(4\log x - 1\right) + \text{C}x^{-2}$$

Question 7:

$$x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$$

Answer

The given differential equation is:

$$x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x^2}$$

This equation is the form of a linear differential equation as:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{1}{x\log x} \text{ and } Q = \frac{2}{x^2})$$

$$\text{Now, I.F } = e^{\int p dx} = e^{\int \frac{1}{x\log} dx} = e^{\log(\log x)} = \log x.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y \log x = \int \left(\frac{2}{x^2} \log x \right) dx + \text{C} \qquad ...(1)$$

Now, $\int \left(\frac{2}{x^2} \log x \right) dx = 2 \int \left(\log x \cdot \frac{1}{x^2} \right) dx.$

$$= 2\left[\log x \cdot \int \frac{1}{x^2}\,dx - \int \left\{ \frac{d}{dx}(\log x) \cdot \int \frac{1}{x^2}\,dx \right\} dx \right]$$

$$= 2\left[\log x \left(-\frac{1}{x} \right) - \int \left(\frac{1}{x} \cdot \left(-\frac{1}{x} \right) \right) dx \right]$$

$$= 2\left[-\frac{\log x}{x} + \int \frac{1}{x^2} dx \right]$$

$$= 2\left[-\frac{\log x}{x} - \frac{1}{x} \right]$$

$$= -\frac{2}{x}(1 + \log x)$$

Substituting the value of $\int \left(\frac{2}{x^2} \log x \right) dx$ in equation (1), we get:

$$y \log x = -\frac{2}{x}(1 + \log x) + \text{C}$$

This is the required general solution of the given differential equation.

Question 8:

$$\left(1 + x^2\right) dy + 2xy\ dx = \cot x dx \left(x \neq 0\right)$$

Answer

$$\left(1 + x^2\right) dy + 2xy\ dx = \cot x dx$$

$$\Rightarrow \frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{\cot x}{1 + x^2}$$

This equation is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{2x}{1 + x^2} \text{ and } Q = \frac{\cot x}{1 + x^2})$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log\left(1 + x^2\right)} = 1 + x^2.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y(1+x^2) = \int \left[\frac{\cot x}{1+x^2} \times (1+x^2)\right] dx + \text{C}$$

$$\Rightarrow y(1+x^2) = \int \cot x\,dx + \text{C}$$

$$\Rightarrow y(1+x^2) = \log|\sin x| + \text{C}$$

Question 9:

$$x\frac{dy}{dx} + y - x + xy\cot x = 0\,(x \neq 0)$$

Answer

$$x\frac{dy}{dx} + y - x + xy\cot x = 0$$

$$\Rightarrow x\frac{dy}{dx} + y(1 + x\cot x) = x$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right) y = 1$$

This equation is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{1}{x} + \cot x \text{ and } Q = 1)$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log x + \log(\sin x)} = e^{\log(x \sin x)} = x\sin x.$$

The general solution of the given differential equation is given by the relation,

$y(\text{I.F.}) = \int(\text{Q} \times \text{I.F.})\,dx + \text{C}$

$\Rightarrow y(x\sin x) = \int(1 \times x\sin x)\,dx + \text{C}$

$\Rightarrow y(x\sin x) = \int(x\sin x)\,dx + \text{C}$

$\Rightarrow y(x\sin x) = x\int\sin x\,dx - \int\left[\frac{d}{dx}(x)\cdot\int\sin x\,dx\right] + \text{C}$

$\Rightarrow y(x\sin x) = x(-\cos x) - \int 1\cdot(-\cos x)\,dx + \text{C}$

$\Rightarrow y(x\sin x) = -x\cos x + \sin x + \text{C}$

$\Rightarrow y = \frac{-x\cos x}{x\sin x} + \frac{\sin x}{x\sin x} + \frac{\text{C}}{x\sin x}$

$\Rightarrow y = -\cot\cdot x + \frac{1}{x} + \frac{\text{C}}{x\sin x}$

Question 10:

$(x+y)\frac{dy}{dx} = 1$

Answer

$(x+y)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$

$\Rightarrow \frac{dx}{dy} = x + y$

$\Rightarrow \frac{dx}{dy} - x = y$

This is a linear differential equation of the form:

$\frac{dy}{dx} + px = Q$ (where $p = -1$ and $Q = y$)

Now, I.F $= e^{\int p\,dy} = e^{\int -dy} = e^{-y}$.

The general solution of the given differential equation is given by the relation,

$$x(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\, dy + \text{C}$$

$$\Rightarrow xe^{-y} = \int \left(y \cdot e^{-y}\right) dy + \text{C}$$

$$\Rightarrow xe^{-y} = y \cdot \int e^{-y} dy - \int \left[\frac{d}{dy}(y) \int e^{-y} dy\right] dy + \text{C}$$

$$\Rightarrow xe^{-y} = y\left(-e^{-y}\right) - \int \left(-e^{-y}\right) dy + \text{C}$$

$$\Rightarrow xe^{-y} = -ye^{-y} + \int e^{-y} dy + \text{C}$$

$$\Rightarrow xe^{-y} = -ye^{-y} - e^{-y} + \text{C}$$

$$\Rightarrow x = -y - 1 + \text{C}e^{y}$$

$$\Rightarrow x + y + 1 = \text{C}e^{y}$$

Question 11:

$$y\, dx + \left(x - y^2\right) dy = 0$$

Answer

$$y\, dx + \left(x - y^2\right) dy = 0$$

$$\Rightarrow ydx = \left(y^2 - x\right) dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{y^2 - x}{y} = y - \frac{x}{y}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y} = y$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + px = Q \text{ (where } p = \frac{1}{y} \text{ and } Q = y)$$

$$\text{Now, I.F } = e^{\int p\,dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y.$$

The general solution of the given differential equation is given by the relation,

$$x(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\,dy + \text{C}$$
$$\Rightarrow xy = \int (y \cdot y)\,dy + \text{C}$$
$$\Rightarrow xy = \int y^2 dy + \text{C}$$
$$\Rightarrow xy = \frac{y^3}{3} + \text{C}$$
$$\Rightarrow x = \frac{y^2}{3} + \frac{\text{C}}{y}$$

Question 12:

$$\left(x + 3y^2\right)\frac{dy}{dx} = y\left(y > 0\right)$$

Answer

$$\left(x + 3y^2\right)\frac{dy}{dx} = y$$
$$\Rightarrow \frac{dy}{dx} = \frac{y}{x + 3y^2}$$
$$\Rightarrow \frac{dx}{dy} = \frac{x + 3y^2}{y} = \frac{x}{y} + 3y$$
$$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$$

This is a linear differential equation of the form:

$$\frac{dx}{dy} + px = Q \text{ (where } p = -\frac{1}{y} \text{ and } Q = 3y)$$

$$\text{Now, I.F } = e^{\int p\,dy} = e^{-\int \frac{dy}{y}} = e^{-\log y} = e^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}.$$

The general solution of the given differential equation is given by the relation,

$$x(\text{I.F.}) = \int(\text{Q}\times\text{I.F.})\,dy + \text{C}$$

$$\Rightarrow x\times\frac{1}{y} = \int\left(3y\times\frac{1}{y}\right)dy + \text{C}$$

$$\Rightarrow \frac{x}{y} = 3y + \text{C}$$

$$\Rightarrow x = 3y^2 + \text{C}y$$

Question 13:

$$\frac{dy}{dx} + 2y\tan x = \sin x;\ y = 0 \text{ when } x = \frac{\pi}{3}$$

Answer

The given differential equation is $\frac{dy}{dx} + 2y\tan x = \sin x.$

This is a linear equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = 2\tan x \text{ and } Q = \sin x)$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int 2\tan x\,dx} = e^{2\log|\sec x|} = e^{\log\left(\sec^2 x\right)} = \sec^2 x.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int(\text{Q}\times\text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y\left(\sec^2 x\right) = \int\left(\sin x\cdot\sec^2 x\right)dx + \text{C}$$

$$\Rightarrow y\sec^2 x = \int(\sec x\cdot\tan x)\,dx + \text{C}$$

$$\Rightarrow y\sec^2 x = \sec x + \text{C} \qquad ...(1)$$

Now, $y = 0 \text{ at } x = \frac{\pi}{3}.$

Therefore,

$$0\times\sec^2\frac{\pi}{3} = \sec\frac{\pi}{3} + \text{C}$$

$$\Rightarrow 0 = 2 + \text{C}$$

$$\Rightarrow \text{C} = -2$$

Substituting C = –2 in equation (1), we get:

$$y\sec^2 x = \sec x - 2$$
$$\Rightarrow y = \cos x - 2\cos^2 x$$

Hence, the required solution of the given differential equation is $y = \cos x - 2\cos^2 x.$

Question 14:

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}; y = 0 \text{ when } x = 1$$

Answer

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$
$$\Rightarrow \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{\left(1+x^2\right)^2}$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{2x}{1+x^2} \text{ and } Q = \frac{1}{\left(1+x^2\right)^2})$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{\int \frac{2x\,dx}{1+x^2}} = e^{\log\left(1+x^2\right)} = 1+x^2.$$

The general solution of the given differential equation is given by the relation,

$$y\left(\text{I.F.}\right) = \int\left(\text{Q} \times \text{I.F.}\right)dx + \text{C}$$
$$\Rightarrow y\left(1+x^2\right) = \int\left[\frac{1}{\left(1+x^2\right)^2}\cdot\left(1+x^2\right)\right]dx + \text{C}$$
$$\Rightarrow y\left(1+x^2\right) = \int\frac{1}{1+x^2}dx + \text{C}$$
$$\Rightarrow y\left(1+x^2\right) = \tan^{-1} x + \text{C} \qquad ...(1)$$

Now, $y = 0$ at $x = 1$.

Therefore,

$$0 = \tan^{-1} 1 + \text{C}$$
$$\Rightarrow \text{C} = -\frac{\pi}{4}$$

Substituting $C = -\frac{\pi}{4}$ in equation (1), we get:

$$y\left(1+x^2\right) = \tan^{-1} x - \frac{\pi}{4}$$

This is the required general solution of the given differential equation.

Question 15:

$$\frac{dy}{dx} - 3y\cot x = \sin 2x;\ y = 2 \text{ when } x = \frac{\pi}{2}$$

Answer

The given differential equation is $\frac{dy}{dx} - 3y\cot x = \sin 2x.$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = -3\cot x \text{ and } Q = \sin 2x)$$

$$\text{Now, I.F } = e^{\int p\,dx} = e^{-3\int \cot x\,dx} = e^{-3\log|\sin x|} = e^{\log\left|\frac{1}{\sin^3 x}\right|} = \frac{1}{\sin^3 x}.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int(\text{Q} \times \text{I.F.})\,dx + \text{C}$$

$$\Rightarrow y \cdot \frac{1}{\sin^3 x} = \int\left[\sin 2x.\frac{1}{\sin^3 x}\right]dx + \text{C}$$

$$\Rightarrow y\,\text{cosec}^3 x = 2\int(\cot x \,\text{cosec} x)\,dx + \text{C}$$

$$\Rightarrow y\,\text{cosec}^3 x = 2\text{cosec } x + \text{C}$$

$$\Rightarrow y = -\frac{2}{\text{cosec}^2 x} + \frac{3}{\text{cosec}^3 x}$$

$$\Rightarrow y = -2\sin^2 x + \text{C}\sin^3 x \qquad ...(1)$$

Now, $y = 2 \text{ at } x = \frac{\pi}{2}.$

Therefore, we get:

$$2 = -2 + \text{C}$$

$$\Rightarrow \text{C} = 4$$

Substituting C = 4 in equation (1), we get:

$y = -2\sin^2 x + 4\sin^3 x$

$\Rightarrow y = 4\sin^3 x - 2\sin^2 x$

This is the required particular solution of the given differential equation.

Question 16:

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

Answer

Let F (x, y) be the curve passing through the origin.

At point (x, y), the slope of the curve will be $\frac{dy}{dx}$.

According to the given information:

$\frac{dy}{dx} = x + y$

$\Rightarrow \frac{dy}{dx} - y = x$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ (where $p = -1$ and $Q = x$)

Now, I.F $= e^{\int p\,dx} = e^{\int(-1)dx} = e^{-x}$.

The general solution of the given differential equation is given by the relation,

$y(\text{I.F.}) = \int(\text{Q} \times \text{I.F.})\,dx + \text{C}$

$\Rightarrow ye^{-x} = \int xe^{-x}dx + \text{C}$ ...(1)

Now, $\int xe^{-x}dx = x\int e^{-x}dx - \int\left[\frac{d}{dx}(x)\cdot\int e^{-x}dx\right]dx.$

$= -xe^{-x} - \int -e^{-x}dx$

$= -xe^{-x} + \left(-e^{-x}\right)$

$= -e^{-x}(x+1)$

Substituting in equation (1), we get:

$$ye^{-x} = -e^{-x}(x+1) + C$$

$$\Rightarrow y = -(x+1) + Ce^{x}$$

$$\Rightarrow x + y + 1 = Ce^{x} \quad \ldots(2)$$

The curve passes through the origin.

Therefore, equation (2) becomes:

1 = C

⇒ C = 1

Substituting C = 1 in equation (2), we get:

$$x + y + 1 = e^{x}$$

Hence, the required equation of curve passing through the origin is $x + y + 1 = e^{x}$.

Question 17:

Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Answer

Let *F* (*x*, *y*) be the curve and let (*x*, *y*) be a point on the curve. The slope of the tangent to the curve at (*x*, *y*) is $\frac{dy}{dx}$.

According to the given information:

$$\frac{dy}{dx} + 5 = x + y$$

$$\Rightarrow \frac{dy}{dx} - y = x - 5$$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ (where $p = -1$ and $Q = x - 5$)

Now, I.F $= e^{\int p dx} = e^{\int (-1) dx} = e^{-x}$.

The general equation of the curve is given by the relation,

$$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.}) dx + \text{C}$$

$$\Rightarrow y \cdot e^{-x} = \int (x-5) e^{-x} dx + \text{C} \quad ...(1)$$

Now, $\int (x-5) e^{-x} dx = (x-5) \int e^{-x} dx - \int \left[\frac{d}{dx}(x-5) . \int e^{-x} dx \right] dx.$

$$= (x-5)(-e^{-x}) - \int (-e^{-x}) dx$$

$$= (5-x)e^{-x} + (-e^{-x})$$

$$= (4-x)e^{-x}$$

Therefore, equation (1) becomes:

$$ye^{-x} = (4-x)e^{-x} + \text{C}$$

$$\Rightarrow y = 4 - x + \text{C}e^{x}$$

$$\Rightarrow x + y - 4 = \text{C}e^{x} \quad ...(2)$$

The curve passes through point (0, 2).

Therefore, equation (2) becomes:

$0 + 2 - 4 = \text{C}e^{0}$

$\Rightarrow -2 = \text{C}$

$\Rightarrow \text{C} = -2$

Substituting C = –2 in equation (2), we get:

$$x + y - 4 = -2e^{x}$$

$$\Rightarrow y = 4 - x - 2e^{x}$$

This is the required equation of the curve.

Question 18:

The integrating factor of the differential equation $x\frac{dy}{dx}-y=2x^2$ is

A. e^{-x}

B. e^{-y}

C. $\frac{1}{x}$

D. x

Answer

The given differential equation is:

$$x\frac{dy}{dx}-y=2x^2$$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=2x$$

This is a linear differential equation of the form:

$$\frac{dy}{dx}+py=Q \text{ (where } p=-\frac{1}{x} \text{ and } Q=2x)$$

The integrating factor (I.F) is given by the relation,

$$e^{\int p dx}$$

$$\therefore \text{I.F } =e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\left(x^{-1}\right)}=x^{-1}=\frac{1}{x}$$

Hence, the correct answer is C.

Question 19:

The integrating factor of the differential equation.

$$\left(1-y^2\right)\frac{dx}{dy}+yx=ay\left(-1<y<1\right)$$ is

A. $\frac{1}{y^2-1}$

B. $\frac{1}{\sqrt{y^2-1}}$

C. $\frac{1}{1-y^2}$

D. $\frac{1}{\sqrt{1-y^2}}$

Answer

The given differential equation is:

$$\left(1-y^2\right)\frac{dx}{dy}+yx=ay$$

$$\Rightarrow \frac{dy}{dx}+\frac{yx}{1-y^2}=\frac{ay}{1-y^2}$$

This is a linear differential equation of the form:

$$\frac{dx}{dy}+py=Q \text{ (where } p=\frac{y}{1-y^2} \text{ and } Q=\frac{ay}{1-y^2})$$

The integrating factor (I.F) is given by the relation,

$$e^{\int pdx}$$

$$\therefore \text{I.F } = e^{\int pdy} = e^{\int \frac{y}{1-y^2}dy} = e^{-\frac{1}{2}\log\left(1-y^2\right)} = e^{\log\left[\frac{1}{\sqrt{1-y^2}}\right]} = \frac{1}{\sqrt{1-y^2}}$$

Hence, the correct answer is D.