

Exercise 9.3**Question 1:**

Find the 20th and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$

Answer

The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$

Here, a = First term = $\frac{5}{2}$

r = Common ratio = $\frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

$$a_{20} = ar^{20-1} = \frac{5}{2} \left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$

$$a_n = ar^{n-1} = \frac{5}{2} \left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

Question 2:

Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Answer

Common ratio, $r = 2$

Let a be the first term of the G.P.

$$\therefore a_8 = ar^{8-1} = ar^7$$

$$\Rightarrow ar^7 = 192$$

$$a(2)^7 = 192$$

$$a(2)^7 = (2)^6 (3)$$

$$\Rightarrow a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$$

$$\therefore a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

Question 3:

The 5th, 8th and 11th terms of a G.P. are p , q and s , respectively. Show that $q^2 = ps$.

Answer

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$$a_5 = ar^{5-1} = ar^4 = p \dots (1)$$

$$a_8 = ar^{8-1} = ar^7 = q \dots (2)$$

$$a_{11} = ar^{11-1} = ar^{10} = s \dots (3)$$

Dividing equation (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \quad \dots(4)$$

Dividing equation (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$\Rightarrow r^3 = \frac{s}{q} \quad \dots(5)$$

Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Thus, the given result is proved.

Question 4:

The 4th term of a G.P. is square of its second term, and the first term is -3 . Determine its 7th term.

Answer

Let a be the first term and r be the common ratio of the G.P.

$$\therefore a = -3$$

It is known that, $a_n = ar^{n-1}$

$$\therefore a_4 = ar^3 = (-3) r^3$$

$$a_2 = ar^1 = (-3) r$$

According to the given condition,

$$(-3) r^3 = [(-3) r]^2$$

$$\Rightarrow -3r^3 = 9 r^2$$

$$\Rightarrow r = -3$$

$$a_7 = ar^{7-1} = ar^6 = (-3) (-3)^6 = - (3)^7 = -2187$$

Thus, the seventh term of the G.P. is -2187 .

Question 5:

Which term of the following sequences:

(a) $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729? (b) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$ is $\frac{1}{19683}$?

Answer

(a) The given sequence is $2, 2\sqrt{2}, 4, \dots$

$$\text{Here, } a = 2 \text{ and } r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Let the n^{th} term of the given sequence be 128.

$$\begin{aligned}
 a_n &= ar^{n-1} \\
 \Rightarrow (2)(\sqrt{2})^{n-1} &= 128 \\
 \Rightarrow (2)(2)^{\frac{n-1}{2}} &= (2)^7 \\
 \Rightarrow (2)^{\frac{n-1}{2}+1} &= (2)^7 \\
 \therefore \frac{n-1}{2} + 1 &= 7 \\
 \Rightarrow \frac{n-1}{2} &= 6 \\
 \Rightarrow n-1 &= 12 \\
 \Rightarrow n &= 13
 \end{aligned}$$

Thus, the 13th term of the given sequence is 128.

(b) The given sequence is $\sqrt{3}, 3, 3\sqrt{3}, \dots$

$$a = \sqrt{3} \text{ and } r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Here,

Let the n^{th} term of the given sequence be 729.

$$\begin{aligned}
 a_n &= ar^{n-1} \\
 \therefore ar^{n-1} &= 729 \\
 \Rightarrow (\sqrt{3})(\sqrt{3})^{n-1} &= 729 \\
 \Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} &= (3)^6 \\
 \Rightarrow (3)^{\frac{1+n-1}{2}} &= (3)^6 \\
 \therefore \frac{1}{2} + \frac{n-1}{2} &= 6 \\
 \Rightarrow \frac{1+n-1}{2} &= 6 \\
 \Rightarrow n &= 12
 \end{aligned}$$

Thus, the 12th term of the given sequence is 729.

(c) The given sequence is $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

Here, $a = \frac{1}{3}$ and $r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$

Let the n^{th} term of the given sequence be $\frac{1}{19683}$.

$$a_n = ar^{n-1}$$

$$\therefore ar^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9$$

$$\Rightarrow n = 9$$

Thus, the 9th term of the given sequence is $\frac{1}{19683}$.

Question 6:

For what values of x , the numbers $\frac{2}{7}, x, -\frac{7}{2}$ are in G.P?

Answer

The given numbers are $\frac{-2}{7}, x, \frac{-7}{2}$.

$$\text{Common ratio} = \frac{\frac{x}{-2}}{\frac{-2}{7}} = \frac{-7x}{2}$$

$$\text{Also, common ratio} = \frac{\frac{-7}{2}}{x} = \frac{-7}{2x}$$

$$\begin{aligned} \therefore \frac{-7x}{2} &= \frac{-7}{2x} \\ \Rightarrow x^2 &= \frac{-2 \times 7}{-2 \times 7} = 1 \\ \Rightarrow x &= \sqrt{1} \\ \Rightarrow x &= \pm 1 \end{aligned}$$

Thus, for $x = \pm 1$, the given numbers will be in G.P.

Question 7:

Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 ...

Answer

The given G.P. is 0.15, 0.015, 0.00015, ...

Here, $a = 0.15$ and $r = \frac{0.015}{0.15} = 0.1$

$$\begin{aligned} S_n &= \frac{a(1-r^n)}{1-r} \\ \therefore S_{20} &= \frac{0.15[1-(0.1)^{20}]}{1-0.1} \\ &= \frac{0.15}{0.9}[1-(0.1)^{20}] \\ &= \frac{15}{90}[1-(0.1)^{20}] \\ &= \frac{1}{6}[1-(0.1)^{20}] \end{aligned}$$

Question 8:

Find the sum to n terms in the geometric progression $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots$

Answer

The given G.P. is $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots$

Here, $a = \sqrt{7}$

$$\begin{aligned}
 r &= \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3} \\
 S_n &= \frac{a(1-r^n)}{1-r} \\
 \therefore S_n &= \frac{\sqrt{7} [1 - (\sqrt{3})^n]}{1 - \sqrt{3}} \\
 &= \frac{\sqrt{7} [1 - (\sqrt{3})^n]}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \quad \text{(By rationalizing)} \\
 &= \frac{\sqrt{7}(1 + \sqrt{3}) [1 - (\sqrt{3})^n]}{1 - 3} \\
 &= \frac{-\sqrt{7}(1 + \sqrt{3}) [1 - (3)^{\frac{n}{2}}]}{2} \\
 &= \frac{\sqrt{7}(1 + \sqrt{3}) [(3)^{\frac{n}{2}} - 1]}{2}
 \end{aligned}$$

Question 9:

Find the sum to n terms in the geometric progression $1, -a, a^2, -a^3 \dots$ (if $a \neq -1$)

Answer

The given G.P. is $1, -a, a^2, -a^3, \dots$

Here, first term = $a_1 = 1$

Common ratio = $r = -a$

$$\begin{aligned}
 S_n &= \frac{a_1(1-r^n)}{1-r} \\
 \therefore S_n &= \frac{1[1 - (-a)^n]}{1 - (-a)} = \frac{[1 - (-a)^n]}{1 + a}
 \end{aligned}$$

Question 10:

Find the sum to n terms in the geometric progression $x^3, x^5, x^7 \dots$ (if $x \neq \pm 1$)

Answer

The given G.P. is x^3, x^5, x^7, \dots

Here, $a = x^3$ and $r = x^2$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3[1-(x^2)^n]}{1-x^2} = \frac{x^3(1-x^{2n})}{1-x^2}$$

Question 11:

Evaluate $\sum_{k=1}^{11} (2+3^k)$

Answer

$$\sum_{k=1}^{11} (2+3^k) = \sum_{k=1}^{11} (2) + \sum_{k=1}^{11} 3^k = 2(11) + \sum_{k=1}^{11} 3^k = 22 + \sum_{k=1}^{11} 3^k \quad \dots(1)$$

$$\sum_{k=1}^{11} 3^k = 3^1 + 3^2 + 3^3 + \dots + 3^{11}$$

The terms of this sequence $3, 3^2, 3^3, \dots$ forms a G.P.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{11} = \frac{3[(3)^{11} - 1]}{3 - 1}$$

$$\Rightarrow S_{11} = \frac{3}{2}(3^{11} - 1)$$

$$\therefore \sum_{k=1}^{11} 3^k = \frac{3}{2}(3^{11} - 1)$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11} (2+3^k) = 22 + \frac{3}{2}(3^{11} - 1)$$

Question 12:

The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Answer

Let $\frac{a}{r}, a, ar$ be the first three terms of the G.P.

$$\frac{a}{r} + a + ar = \frac{39}{10} \quad \dots(1)$$

$$\left(\frac{a}{r}\right)(a)(ar) = 1 \quad \dots(2)$$

From (2), we obtain

$$a^3 = 1$$

$\Rightarrow a = 1$ (Considering real roots only)

Substituting $a = 1$ in equation (1), we obtain

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$\Rightarrow 1 + r + r^2 = \frac{39}{10}r$$

$$\Rightarrow 10 + 10r + 10r^2 - 39r = 0$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$$

$$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$$

$$\Rightarrow (5r - 2)(2r - 5) = 0$$

$$\Rightarrow r = \frac{2}{5} \text{ or } \frac{5}{2}$$

Thus, the three terms of G.P. are $\frac{5}{2}, 1, \text{ and } \frac{2}{5}$.

Question 13:

How many terms of G.P. $3, 3^2, 3^3, \dots$ are needed to give the sum 120?

Answer

The given G.P. is $3, 3^2, 3^3, \dots$

Let n terms of this G.P. be required to obtain the sum as 120.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Here, $a = 3$ and $r = 3$

$$\begin{aligned}\therefore S_n = 120 &= \frac{3(3^n - 1)}{3 - 1} \\ \Rightarrow 120 &= \frac{3(3^n - 1)}{2} \\ \Rightarrow \frac{120 \times 2}{3} &= 3^n - 1 \\ \Rightarrow 3^n - 1 &= 80 \\ \Rightarrow 3^n &= 81 \\ \Rightarrow 3^n &= 3^4 \\ \therefore n &= 4\end{aligned}$$

Thus, four terms of the given G.P. are required to obtain the sum as 120.

Question 14:

The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Answer

Let the G.P. be a, ar, ar^2, ar^3, \dots

According to the given condition,

$$a + ar + ar^2 = 16 \text{ and } ar^3 + ar^4 + ar^5 = 128$$

$$\Rightarrow a(1 + r + r^2) = 16 \dots (1)$$

$$ar^3(1 + r + r^2) = 128 \dots (2)$$

Dividing equation (2) by (1), we obtain

$$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$$

$$\Rightarrow r^3 = 8$$

$$\therefore r = 2$$

Substituting $r = 2$ in (1), we obtain

$$a(1 + 2 + 4) = 16$$

$$\Rightarrow a(7) = 16$$

$$\Rightarrow a = \frac{16}{7}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{16(2^n - 1)}{7 \cdot 2 - 1} = \frac{16}{7}(2^n - 1)$$

Question 15:

Given a G.P. with $a = 729$ and 7th term 64, determine S_7 .

Answer

$$a = 729$$

$$a_7 = 64$$

Let r be the common ratio of the G.P.

It is known that, $a_n = a r^{n-1}$

$$a_7 = ar^{7-1} = (729)r^6$$

$$\Rightarrow 64 = 729 r^6$$

$$\Rightarrow r^6 = \frac{64}{729}$$

$$\Rightarrow r^6 = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow r = \frac{2}{3}$$

Also, it is known that, $S_n = \frac{a(1-r^n)}{1-r}$

$$\begin{aligned}\therefore S_7 &= \frac{729 \left[1 - \left(\frac{2}{3} \right)^7 \right]}{1 - \frac{2}{3}} \\ &= 3 \times 729 \left[1 - \left(\frac{2}{3} \right)^7 \right] \\ &= (3)^7 \left[\frac{(3)^7 - (2)^7}{(3)^7} \right] \\ &= (3)^7 - (2)^7 \\ &= 2187 - 128 \\ &= 2059\end{aligned}$$

Question 16:

Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

Answer

Let a be the first term and r be the common ratio of the G.P.

According to the given conditions,

$$S_2 = -4 = \frac{a(1-r^2)}{1-r} \quad \dots(1)$$

$$a_5 = 4 \times a_3$$

$$ar^4 = 4ar^2$$

$$\Rightarrow r^2 = 4$$

$$\therefore r = \pm 2$$

From (1), we obtain

$$-4 = \frac{a[1-(2)^2]}{1-2} \text{ for } r = 2$$

$$\Rightarrow -4 = \frac{a(1-4)}{-1}$$

$$\Rightarrow -4 = a(3)$$

$$\Rightarrow a = \frac{-4}{3}$$

$$\text{Also, } -4 = \frac{a[1-(-2)^2]}{1-(-2)} \text{ for } r = -2$$

$$\Rightarrow -4 = \frac{a(1-4)}{1+2}$$

$$\Rightarrow -4 = \frac{a(-3)}{3}$$

$$\Rightarrow a = 4$$

Thus, the required G.P. is

$$\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \dots \text{ or } 4, -8, 16, -32, \dots$$

Question 17:

If the 4th, 10th and 16th terms of a G.P. are x , y and z , respectively. Prove that x , y , z are in G.P.

Answer

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$$a_4 = a r^3 = x \dots (1)$$

$$a_{10} = a r^9 = y \dots (2)$$

$$a_{16} = a r^{15} = z \dots (3)$$

Dividing (2) by (1), we obtain

$$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$$

Dividing (3) by (2), we obtain

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \Rightarrow \frac{z}{y} = r^6$$

$$\therefore \frac{y}{x} = \frac{z}{y}$$

Thus, x, y, z are in G. P.

Question 18:

Find the sum to n terms of the sequence, 8, 88, 888, 8888...

Answer

The given sequence is 8, 88, 888, 8888...

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

$S_n = 8 + 88 + 888 + 8888 + \dots$ to n terms

$$\begin{aligned} &= \frac{8}{9} [9 + 99 + 999 + 9999 + \dots \text{to } n \text{ terms}] \\ &= \frac{8}{9} [(10-1) + (10^2-1) + (10^3-1) + (10^4-1) + \dots \text{to } n \text{ terms}] \\ &= \frac{8}{9} [(10 + 10^2 + \dots \text{to } n \text{ terms}) - (1 + 1 + 1 + \dots \text{to } n \text{ terms})] \\ &= \frac{8}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right] \\ &= \frac{8}{9} \left[\frac{10(10^n - 1)}{9} - n \right] \\ &= \frac{80}{81} (10^n - 1) - \frac{8}{9} n \end{aligned}$$

Question 19:

Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32

and 128, 32, 8, 2, $\frac{1}{2}$.

Answer

$$\text{Required sum} = 2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$$

$$= 64 \left[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2} \right]$$

Here, $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}$ is a G.P.

First term, $a = 4$

Common ratio, $r = \frac{1}{2}$

It is known that, $S_n = \frac{a(1-r^n)}{1-r}$

$$\therefore S_5 = \frac{4 \left[1 - \left(\frac{1}{2} \right)^5 \right]}{1 - \frac{1}{2}} = \frac{4 \left[1 - \frac{1}{32} \right]}{\frac{1}{2}} = 8 \left(\frac{32-1}{32} \right) = \frac{31}{4}$$

$$\therefore \text{Required sum} = 64 \left(\frac{31}{4} \right) = (16)(31) = 496$$

Question 20:

Show that the products of the corresponding terms of the sequences

$a, ar, ar^2, \dots, ar^{n-1}$ and $A, AR, AR^2, \dots, AR^{n-1}$ form a G.P, and find the common ratio.

Answer

It has to be proved that the sequence, $aA, arAR, ar^2AR^2, \dots, ar^{n-1}AR^{n-1}$, forms a G.P.

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$

$$\frac{\text{Third term}}{\text{Second term}} = \frac{ar^2AR^2}{arAR} = rR$$

Thus, the above sequence forms a G.P. and the common ratio is rR .

Question 21:

Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Answer

Let a be the first term and r be the common ratio of the G.P.

$$a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$$

By the given condition,

$$a_3 = a_1 + 9$$

$$\Rightarrow ar^2 = a + 9 \dots (1)$$

$$a_2 = a_4 + 18$$

$$\Rightarrow ar = ar^3 + 18 \dots (2)$$

From (1) and (2), we obtain

$$a(r^2 - 1) = 9 \dots (3)$$

$$ar(1 - r^2) = 18 \dots (4)$$

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$

$$\Rightarrow -r = 2$$

$$\Rightarrow r = -2$$

Substituting the value of r in (1), we obtain

$$4a = a + 9$$

$$\Rightarrow 3a = 9$$

$$\therefore a = 3$$

Thus, the first four numbers of the G.P. are 3, $3(-2)$, $3(-2)^2$, and $3(-2)^3$ i.e., 3, -6, 12, and -24.

Question 22:

If the p^{th} , q^{th} and r^{th} terms of a G.P. are a , b and c , respectively. Prove that

$$a^{q-r} b^{r-p} c^{p-q} = 1$$

Answer

Let A be the first term and R be the common ratio of the G.P.

According to the given information,

$$AR^{p-1} = a$$

$$AR^{q-1} = b$$

$$AR^{r-1} = c$$

$$a^{q-r} b^{r-p} c^{p-q}$$

$$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$$

$$= Aq^{-r+r-p+p-q} \times R^{(pr-pr-q+r) + (rq-r+p-pq) + (pr-p-qr+q)}$$

$$= A^0 \times R^0$$

$$= 1$$

Thus, the given result is proved.

Question 23:

If the first and the n^{th} term of a G.P. are a and b , respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Answer

The first term of the G.P is a and the last term is b .

Therefore, the G.P. is $a, ar, ar^2, ar^3, \dots, ar^{n-1}$, where r is the common ratio.

$$b = ar^{n-1} \dots (1)$$

$P =$ Product of n terms

$$= (a) (ar) (ar^2) \dots (ar^{n-1})$$

$$= (a \times a \times \dots a) (r \times r^2 \times \dots r^{n-1})$$

$$= a^n r^{1+2+\dots+(n-1)} \dots (2)$$

Here, $1, 2, \dots, (n-1)$ is an A.P.

$$\therefore 1 + 2 + \dots + (n-1) = \frac{n-1}{2} [2 + (n-1-1) \times 1] = \frac{n-1}{2} [2 + n - 2] = \frac{n(n-1)}{2}$$

$$P = a^n r^{\frac{n(n-1)}{2}}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$= [a^2 r^{(n-1)}]^n$$

$$= [a \times ar^{n-1}]^n$$

$$= (ab)^n \quad [\text{Using (1)}]$$

Thus, the given result is proved.

Question 24:

Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from

$$(n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term is } \frac{1}{r^n}.$$

Answer

Let a be the first term and r be the common ratio of the G.P.

$$\text{Sum of first } n \text{ terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are n terms from $(n+1)^{\text{th}}$ to $(2n)^{\text{th}}$ term,

$$\text{Sum of terms from } (n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term} = \frac{a_{n+1}(1-r^n)}{(1-r)}$$

$$a^{n+1} = ar^{n+1-1} = ar^n$$

$$\text{Thus, required ratio} = \frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r^n)} = \frac{1}{r^n}$$

Thus, the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{\text{th}}$ to

$$(2n)^{\text{th}} \text{ term is } \frac{1}{r^n}.$$

Question 25:

If a, b, c and d are in G.P. show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

Answer

a, b, c, d are in G.P.

Therefore,

$$bc = ad \dots (1)$$

$$b^2 = ac \dots (2)$$

$$c^2 = bd \dots (3)$$

It has to be proved that,

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$$

R.H.S.

$$= (ab + bc + cd)^2$$

$$\begin{aligned}
&= (ab + ad + cd)^2 \text{ [Using (1)]} \\
&= [ab + d(a + c)]^2 \\
&= a^2b^2 + 2abd(a + c) + d^2(a + c)^2 \\
&= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2) \\
&= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2 \text{ [Using (1) and (2)]} \\
&= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2 \\
&= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2 \\
&\text{[Using (2) and (3) and rearranging terms]} \\
&= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2) \\
&= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \\
&= \text{L.H.S.} \\
&\therefore \text{L.H.S.} = \text{R.H.S.} \\
&\therefore (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2
\end{aligned}$$

Question 26:

Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Answer

Let G_1 and G_2 be two numbers between 3 and 81 such that the series, 3, G_1 , G_2 , 81, forms a G.P.

Let a be the first term and r be the common ratio of the G.P.

$$\therefore 81 = (3)(r)^3$$

$$\Rightarrow r^3 = 27$$

$$\therefore r = 3 \text{ (Taking real roots only)}$$

For $r = 3$,

$$G_1 = ar = (3)(3) = 9$$

$$G_2 = ar^2 = (3)(3)^2 = 27$$

Thus, the required two numbers are 9 and 27.

Question 27:

Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b .

Answer

G. M. of a and b is \sqrt{ab} .

By the given condition, $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$

Squaring both sides, we obtain

$$\begin{aligned} \frac{(a^{n+1} + b^{n+1})^2}{(a^n + b^n)^2} &= ab \\ \Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} &= (ab)(a^{2n} + 2a^n b^n + b^{2n}) \\ \Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} &= a^{2n+1}b + 2a^{n+1}b^{n+1} + ab^{2n+1} \\ \Rightarrow a^{2n+2} + b^{2n+2} &= a^{2n+1}b + ab^{2n+1} \\ \Rightarrow a^{2n+2} - a^{2n+1}b &= ab^{2n+1} - b^{2n+2} \\ \Rightarrow a^{2n+1}(a-b) &= b^{2n+1}(a-b) \\ \Rightarrow \left(\frac{a}{b}\right)^{2n+1} &= 1 = \left(\frac{a}{b}\right)^0 \\ \Rightarrow 2n+1 &= 0 \\ \Rightarrow n &= \frac{-1}{2} \end{aligned}$$

Question 28:

The sum of two numbers is 6 times their geometric mean, show that numbers are in the

ratio $(3+2\sqrt{2}) : (3-2\sqrt{2})$.

Answer

Let the two numbers be a and b .

$$\text{G.M.} = \sqrt{ab}$$

According to the given condition,

$$a + b = 6\sqrt{ab} \quad \dots(1)$$

$$\Rightarrow (a+b)^2 = 36(ab)$$

Also,

$$\begin{aligned}(a-b)^2 &= (a+b)^2 - 4ab = 36ab - 4ab = 32ab \\ \Rightarrow a-b &= \sqrt{32}\sqrt{ab} \\ &= 4\sqrt{2}\sqrt{ab} \quad \dots(2)\end{aligned}$$

Adding (1) and (2), we obtain

$$\begin{aligned}2a &= (6+4\sqrt{2})\sqrt{ab} \\ \Rightarrow a &= (3+2\sqrt{2})\sqrt{ab}\end{aligned}$$

Substituting the value of a in (1), we obtain

$$\begin{aligned}b &= 6\sqrt{ab} - (3+2\sqrt{2})\sqrt{ab} \\ \Rightarrow b &= (3-2\sqrt{2})\sqrt{ab} \\ \frac{a}{b} &= \frac{(3+2\sqrt{2})\sqrt{ab}}{(3-2\sqrt{2})\sqrt{ab}} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}\end{aligned}$$

Thus, the required ratio is $(3+2\sqrt{2}) : (3-2\sqrt{2})$.

Question 29:

If A and G be A.M. and G.M., respectively between two positive numbers, prove that the

numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Answer

It is given that A and G are A.M. and G.M. between two positive numbers. Let these two positive numbers be a and b .

$$\therefore \text{AM} = A = \frac{a+b}{2} \quad \dots(1)$$

$$\text{GM} = G = \sqrt{ab} \quad \dots(2)$$

From (1) and (2), we obtain

$$a + b = 2A \quad \dots (3)$$

$$ab = G^2 \quad \dots (4)$$

Substituting the value of a and b from (3) and (4) in the identity $(a-b)^2 = (a+b)^2 - 4ab$, we obtain

$$(a-b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2)$$

$$(a - b)^2 = 4(A + G)(A - G)$$

$$(a - b) = 2\sqrt{(A + G)(A - G)} \quad \dots(5)$$

From (3) and (5), we obtain

$$2a = 2A + 2\sqrt{(A + G)(A - G)}$$

$$\Rightarrow a = A + \sqrt{(A + G)(A - G)}$$

Substituting the value of a in (3), we obtain

$$b = 2A - A - \sqrt{(A + G)(A - G)} = A - \sqrt{(A + G)(A - G)}$$

Thus, the two numbers are $A \pm \sqrt{(A + G)(A - G)}$.

Question 30:

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and n^{th} hour?

Answer

It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here, $a = 30$ and $r = 2$

$$\therefore a_3 = ar^2 = (30)(2)^2 = 120$$

Therefore, the number of bacteria at the end of 2nd hour will be 120.

$$a_5 = ar^4 = (30)(2)^4 = 480$$

The number of bacteria at the end of 4th hour will be 480.

$$a_{n+1} = ar^n = (30)2^n$$

Thus, number of bacteria at the end of n^{th} hour will be $30(2)^n$.

Question 31:

What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Answer

The amount deposited in the bank is Rs 500.

At the end of first year, amount = $\text{Rs } 500 \left(1 + \frac{1}{10}\right) = \text{Rs } 500 (1.1)$

At the end of 2nd year, amount = $\text{Rs } 500 (1.1) (1.1)$

At the end of 3rd year, amount = $\text{Rs } 500 (1.1) (1.1) (1.1)$ and so on

∴ Amount at the end of 10 years = $\text{Rs } 500 (1.1) (1.1) \dots (10 \text{ times})$

= $\text{Rs } 500(1.1)^{10}$

Question 32:

If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Answer

Let the root of the quadratic equation be a and b .

According to the given condition,

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b = 16 \quad \dots(1)$$

$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \quad \dots(2)$$

The quadratic equation is given by,

$$x^2 - x (\text{Sum of roots}) + (\text{Product of roots}) = 0$$

$$x^2 - x (a + b) + (ab) = 0$$

$$x^2 - 16x + 25 = 0 \text{ [Using (1) and (2)]}$$

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$