Exercise 9.2

Question 1:

Find the sum of odd integers from 1 to 2001.

Answer

The odd integers from 1 to 2001 are 1, 3, 5, …1999, 2001.

This sequence forms an A.P.

Here, first term, a = 1

Common difference, d = 2

$$\text{Here, } a+(n-1)d = 2001$$
$$\Rightarrow 1+(n-1)(2) = 2001$$
$$\Rightarrow 2n-2 = 2000$$
$$\Rightarrow n = 1001$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$\therefore S_n = \frac{1001}{2}\left[2\times1+(1001-1)\times2\right]$$
$$= \frac{1001}{2}\left[2+1000\times2\right]$$
$$= \frac{1001}{2}\times2002$$
$$= 1001\times1001$$
$$= 1002001$$

Thus, the sum of odd numbers from 1 to 2001 is 1002001.

Question 2:

Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Answer

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, … 995.

Here, $a = 105$ and $d = 5$

$a + (n-1)d = 995$

$\Rightarrow 105 + (n-1)5 = 995$

$\Rightarrow (n-1)5 = 995 - 105 = 890$

$\Rightarrow n - 1 = 178$

$\Rightarrow n = 179$

$$\therefore S_n = \frac{179}{2}\left[2(105) + (179-1)(5)\right]$$
$$= \frac{179}{2}\left[2(105) + (178)(5)\right]$$
$$= 179\left[105 + (89)5\right]$$
$$= (179)(105 + 445)$$
$$= (179)(550)$$
$$= 98450$$

Thus, the sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 98450.

Question 3:

In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is –112.

Answer

First term = 2

Let *d* be the common difference of the A.P.

Therefore, the A.P. is 2, 2 + *d*, 2 + 2*d*, 2 + 3*d*, …

Sum of first five terms = 10 + 10*d*

Sum of next five terms = 10 + 35*d*

According to the given condition,

$$10 + 10d = \frac{1}{4}(10 + 35d)$$

$\Rightarrow 40 + 40d = 10 + 35d$

$\Rightarrow 30 = -5d$

$\Rightarrow d = -6$

$\therefore a_{20} = a + (20-1)d = 2 + (19)(-6) = 2 - 114 = -112$

Thus, the 20th term of the A.P. is –112.

Question 4:

How many terms of the A.P. $-6, -\frac{11}{2}, -5, \ldots$ are needed to give the sum –25?

Answer

Let the sum of *n* terms of the given A.P. be –25.

It is known that, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, where *n* = number of terms, *a* = first term, and *d* = common difference

Here, *a* = –6

$d = -\frac{11}{2} + 6 = \frac{-11+12}{2} = \frac{1}{2}$

Therefore, we obtain

$$-25 = \frac{n}{2}\left[2 \times (-6) + (n-1)\left(\frac{1}{2}\right)\right]$$

$$\Rightarrow -50 = n\left[-12 + \frac{n}{2} - \frac{1}{2}\right]$$

$$\Rightarrow -50 = n\left[-\frac{25}{2} + \frac{n}{2}\right]$$

$$\Rightarrow -100 = n(-25 + n)$$

$$\Rightarrow n^2 - 25n + 100 = 0$$

$$\Rightarrow n^2 - 5n - 20n + 100 = 0$$

$$\Rightarrow n(n-5) - 20(n-5) = 0$$

$$\Rightarrow n = 20 \text{ or } 5$$

Question 5:

In an A.P., if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first *pq* terms is $\frac{1}{2}(pq+1)$ where $p \neq q$.

Answer

It is known that the general term of an A.P. is $a_n = a + (n - 1)d$

∴ According to the given information,

$$p^{\text{th}} \text{ term} = a_p = a + (p-1)d = \frac{1}{q} \qquad ...(1)$$

$$q^{\text{th}} \text{ term} = a_q = a + (q-1)d = \frac{1}{p} \qquad ...(2)$$

Subtracting (2) from (1), we obtain

$$(p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$$

$$\Rightarrow (p-1-q+1)d = \frac{p-q}{pq}$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow d = \frac{1}{pq}$$

Putting the value of d in (1), we obtain

$$a + (p-1)\frac{1}{pq} = \frac{1}{q}$$

$$\Rightarrow a = \frac{1}{q} - \frac{1}{q} + \frac{1}{pq} = \frac{1}{pq}$$

$$\therefore S_{pq} = \frac{pq}{2}\left[2a + (pq-1)d\right]$$

$$= \frac{pq}{2}\left[\frac{2}{pq} + (pq-1)\frac{1}{pq}\right]$$

$$= 1 + \frac{1}{2}(pq-1)$$

$$= \frac{1}{2}pq + 1 - \frac{1}{2} = \frac{1}{2}pq + \frac{1}{2}$$

$$= \frac{1}{2}(pq+1)$$

Thus, the sum of first pq terms of the A.P. is $\frac{1}{2}(pq+1)$.

Question 6:

If the sum of a certain number of terms of the A.P. 25, 22, 19, … is 116. Find the last term

Answer

Let the sum of n terms of the given A.P. be 116.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Here, a = 25 and d = 22 – 25 = – 3

$$\therefore S_n = \frac{n}{2}\left[2 \times 25 + (n-1)(-3)\right]$$

$$\Rightarrow 116 = \frac{n}{2}\left[50 - 3n + 3\right]$$

$$\Rightarrow 232 = n(53 - 3n) = 53n - 3n^2$$

$$\Rightarrow 3n^2 - 53n + 232 = 0$$

$$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$$

$$\Rightarrow 3n(n-8) - 29(n-8) = 0$$

$$\Rightarrow (n-8)(3n-29) = 0$$

$$\Rightarrow n = 8 \text{ or } n = \frac{29}{3}$$

However, n cannot be equal to $\frac{29}{3}$. Therefore, n = 8

∴ a_8 = Last term = a + (n – 1)d = 25 + (8 – 1) (– 3)

= 25 + (7) (– 3) = 25 – 21

= 4

Thus, the last term of the A.P. is 4.

Question 7:

Find the sum to n terms of the A.P., whose k^{th} term is 5k + 1.

Answer

It is given that the k^{th} term of the A.P. is 5k + 1.

k^{th} term = a_k = a + (k – 1)d

∴ a + (k – 1)d = 5k + 1

a + kd – d = 5k + 1

Comparing the coefficient of k, we obtain $d = 5$

$a - d = 1$

$\Rightarrow a - 5 = 1$

$\Rightarrow a = 6$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2(6) + (n-1)(5)\right]$$

$$= \frac{n}{2}\left[12 + 5n - 5\right]$$

$$= \frac{n}{2}(5n + 7)$$

Question 8:

If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

Answer

It is known that, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

According to the given condition,

$$\frac{n}{2}\left[2a + (n-1)d\right] = pn + qn^2$$

$$\Rightarrow \frac{n}{2}\left[2a + nd - d\right] = pn + qn^2$$

$$\Rightarrow na + n^2\frac{d}{2} - n\cdot\frac{d}{2} = pn + qn^2$$

Comparing the coefficients of n^2 on both sides, we obtain

$$\frac{d}{2} = q$$

$\therefore d = 2q$

Thus, the common difference of the A.P. is $2q$.

Question 9:

The sums of n terms of two arithmetic progressions are in the ratio $5n + 4: 9n + 6$. Find the ratio of their 18^{th} terms.

Answer

Let a_1, a_2, and d_1, d_2 be the first terms and the common difference of the first and second arithmetic progression respectively.

According to the given condition,

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6} \qquad ...(1)$$

Substituting $n = 35$ in (1), we obtain

$$\frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321} \qquad ...(2)$$

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P}} = \frac{a_1+17d_1}{a_2+17d_2} \qquad ...(3)$$

From (2) and (3), we obtain

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P.}} = \frac{179}{321}$$

Thus, the ratio of 18^{th} term of both the A.P.s is 179: 321.

Question 10:

If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p + q)$ terms.

Answer

Let a and d be the first term and the common difference of the A.P. respectively.

Here,

$$S_p = \frac{p}{2}\left[2a + (p-1)d\right]$$

$$S_q = \frac{q}{2}\left[2a + (q-1)d\right]$$

According to the given condition,

$$\frac{p}{2}\left[2a+(p-1)d\right] = \frac{q}{2}\left[2a+(q-1)d\right]$$
$$\Rightarrow p\left[2a+(p-1)d\right] = q\left[2a+(q-1)d\right]$$
$$\Rightarrow 2ap + pd(p-1) = 2aq + qd(q-1)$$
$$\Rightarrow 2a(p-q) + d\left[p(p-1) - q(q-1)\right] = 0$$
$$\Rightarrow 2a(p-q) + d\left[p^2 - p - q^2 + q\right] = 0$$
$$\Rightarrow 2a(p-q) + d\left[(p-q)(p+q) - (p-q)\right] = 0$$
$$\Rightarrow 2a(p-q) + d\left[(p-q)(p+q-1)\right] = 0$$
$$\Rightarrow 2a + d(p+q-1) = 0$$
$$\Rightarrow d = \frac{-2a}{p+q-1} \quad ...(1)$$
$$\therefore S_{p+q} = \frac{p+q}{2}\left[2a + (p+q-1)\cdot d\right]$$
$$\Rightarrow S_{p+q} = \frac{p+q}{2}\left[2a + (p+q-1)\left(\frac{-2a}{p+q-1}\right)\right] \quad \text{[From (1)]}$$
$$= \frac{p+q}{2}\left[2a - 2a\right]$$
$$= 0$$

Thus, the sum of the first (p + q) terms of the A.P. is 0.

Question 11:

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

Answer

Let a_1 and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$$S_p = \frac{p}{2}\left[2a_1 + (p-1)d\right] = a$$

$$\Rightarrow 2a_1 + (p-1)d = \frac{2a}{p} \qquad ...(1)$$

$$S_q = \frac{q}{2}\left[2a_1 + (q-1)d\right] = b$$

$$\Rightarrow 2a_1 + (q-1)d = \frac{2b}{q} \qquad ...(2)$$

$$S_r = \frac{r}{2}\left[2a_1 + (r-1)d\right] = c$$

$$\Rightarrow 2a_1 + (r-1)d = \frac{2c}{r} \qquad ...(3)$$

Subtracting (2) from (1), we obtain

$$(p-1)d - (q-1)d = \frac{2a}{p} - \frac{2b}{q}$$

$$\Rightarrow d(p-1-q+1) = \frac{2aq-2bq}{pq}$$

$$\Rightarrow d(p-q) = \frac{2aq-2bp}{pq}$$

$$\Rightarrow d = \frac{2(aq-bp)}{pq(p-q)} \qquad ...(4)$$

Subtracting (3) from (2), we obtain

$$(q-1)d - (r-1)d = \frac{2b}{q} - \frac{2c}{r}$$

$$\Rightarrow d(q-1-r+1) = \frac{2b}{q} - \frac{2c}{r}$$

$$\Rightarrow d(q-r) = \frac{2br-2qc}{qr}$$

$$\Rightarrow d = \frac{2(br-qc)}{qr(q-r)} \qquad ...(5)$$

Equating both the values of *d* obtained in (4) and (5), we obtain

$$\frac{aq-bp}{pq(p-q)}=\frac{br-qc}{qr(q-r)}$$

$$\Rightarrow qr(q-r)(aq-bq)=pq(p-q)(br-qc)$$

$$\Rightarrow r(aq-bp)(q-r)=p(br-qc)(p-q)$$

$$\Rightarrow (aqr-bpr)(q-r)=(bpr-pqc)(p-q)$$

Dividing both sides by *pqr*, we obtain

$$\left(\frac{a}{p}-\frac{b}{q}\right)(q-r)=\left(\frac{b}{q}-\frac{c}{r}\right)(p-q)$$

$$\Rightarrow \frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$

$$\Rightarrow \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Thus, the given result is proved.

Question 12:

The ratio of the sums of m and n terms of an A.P. is m^2: n^2. Show that the ratio of m^{th} and n^{th} term is (2*m* – 1): (2*n* – 1).

Answer

Let *a* and *b* be the first term and the common difference of the A.P. respectively.

According to the given condition,

$$\frac{\text{Sum of m terms}}{\text{Sum of n terms}}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n} \qquad ...(1)$$

Putting *m* = 2*m* – 1 and *n* = 2*n* – 1 in (1), we obtain

$$\frac{2a+(2m-2)d}{2a+(2n-2)d}=\frac{2m-1}{2n-1}$$

$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1} \qquad ...(2)$$

$$\frac{m^{th}\text{ term of A.P.}}{n^{th}\text{ term of A.P.}}=\frac{a+(m-1)d}{a+(n-1)d} \qquad ...(3)$$

From (2) and (3), we obtain

$$\frac{m^{th}\text{ term of A.P}}{n^{th}\text{term of A.P}}=\frac{2m-1}{2n-1}$$

Thus, the given result is proved.

Question 13:

If the sum of *n* terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164, find the value of *m*.

Answer

Let *a* and *b* be the first term and the common difference of the A.P. respectively.

$a_m = a + (m - 1)d = 164$ … (1)

Sum of *n* terms, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

Here,

$$\frac{n}{2}\left[2a+nd-d\right]=3n^2+5n$$

$$\Rightarrow na+n^2\cdot\frac{d}{2}=3n^2+5n$$

Comparing the coefficient of n^2 on both sides, we obtain

$$\frac{d}{2}=3$$

$$\Rightarrow d=6$$

Comparing the coefficient of *n* on both sides, we obtain

$$a-\frac{d}{2}=5$$

$$\Rightarrow a-3=5$$

$$\Rightarrow a=8$$

Therefore, from (1), we obtain

$8 + (m - 1)\,6 = 164$

$\Rightarrow (m - 1)\,6 = 164 - 8 = 156$

$\Rightarrow m - 1 = 26$

$\Rightarrow m = 27$

Thus, the value of m is 27.

Question 14:

Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

Answer

Let A_1, A_2, A_3, A_4, and A_5 be five numbers between 8 and 26 such that

8, A_1, A_2, A_3, A_4, A_5, 26 is an A.P.

Here, $a = 8$, $b = 26$, $n = 7$

Therefore, $26 = 8 + (7 - 1)\,d$

$\Rightarrow 6d = 26 - 8 = 18$

$\Rightarrow d = 3$

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Thus, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

Question 15:

If $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

Answer

A.M. of a and b $= \dfrac{a+b}{2}$

According to the given condition,

$$\frac{a+b}{2}=\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$
$$\Rightarrow (a+b)\left(a^{n-1}+b^{n-1}\right)=2\left(a^n+b^n\right)$$
$$\Rightarrow a^n+ab^{n-1}+ba^{n-1}+b^n=2a^n+2b^n$$
$$\Rightarrow ab^{n-1}+a^{n-1}b=a^n+b^n$$
$$\Rightarrow ab^{n-1}-b^n=a^n-a^{n-1}b$$
$$\Rightarrow b^{n-1}(a-b)=a^{n-1}(a-b)$$
$$\Rightarrow b^{n-1}=a^{n-1}$$
$$\Rightarrow \left(\frac{a}{b}\right)^{n-1}=1=\left(\frac{a}{b}\right)^0$$
$$\Rightarrow n-1=0$$
$$\Rightarrow n=1$$

Question 16:

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and $(m - 1)$th numbers is 5:9. Find the value of m.

Answer

Let A_1, A_2, … A_m be m numbers such that 1, A_1, A_2, … A_m, 31 is an A.P.

Here, $a = 1$, $b = 31$, $n = m + 2$

$\therefore 31 = 1 + (m + 2 - 1)(d)$

$\Rightarrow 30 = (m + 1)d$

$$\Rightarrow d=\frac{30}{m+1} \qquad ...(1)$$

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3d$ …

$\therefore A_7 = a + 7d$

$A_{m-1} = a + (m - 1)d$

According to the given condition,

$$\frac{a+7d}{a+(m-1)d}=\frac{5}{9}$$

$$\Rightarrow \frac{1+7\left(\frac{30}{(m+1)}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)}=\frac{5}{9} \qquad \text{[From (1)]}$$

$$\Rightarrow \frac{m+1+7(30)}{m+1+30(m-1)}=\frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30}=\frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29}=\frac{5}{9}$$

$$\Rightarrow 9m+1899=155m-145$$

$$\Rightarrow 155m-9m=1899+145$$

$$\Rightarrow 146m=2044$$

$$\Rightarrow m=14$$

Thus, the value of *m* is 14.

Question 17:

A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs 5 every month, what amount he will pay in the 30th installment?

Answer

The first installment of the loan is Rs 100.

The second installment of the loan is Rs 105 and so on.

The amount that the man repays every month forms an A.P.

The A.P. is 100, 105, 110, …

First term, a = 100

Common difference, d = 5

$A_{30} = a + (30 - 1)d$

= 100 + (29) (5)

= 100 + 145

= 245

Thus, the amount to be paid in the 30th installment is Rs 245.

Question 18:

The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

Answer

The angles of the polygon will form an A.P. with common difference *d* as 5° and first term *a* as 120°.

It is known that the sum of all angles of a polygon with *n* sides is 180° (*n* – 2).

$$\therefore S_n = 180°(n-2)$$

$$\Rightarrow \frac{n}{2}\left[2a+(n-1)d\right] = 180°(n-2)$$

$$\Rightarrow \frac{n}{2}\left[240°+(n-1)5°\right] = 180(n-2)$$

$$\Rightarrow n\left[240+(n-1)5\right] = 360(n-2)$$

$$\Rightarrow 240n+5n^2-5n = 360n-720$$

$$\Rightarrow 5n^2+235n-360n+720 = 0$$

$$\Rightarrow 5n^2-125n+720 = 0$$

$$\Rightarrow n^2-25n+144 = 0$$

$$\Rightarrow n^2-16n-9n+144 = 0$$

$$\Rightarrow n(n-16)-9(n-16) = 0$$

$$\Rightarrow (n-9)(n-16) = 0$$

$$\Rightarrow n = 9 \text{ or } 16$$